SOME HEAT-CONDUCTION PROBLEMS IN THE ELECTROMAGNETIC AND ACOUSTIC INTERACTION WITH THE DIELECTRICS

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We solve the inverse problem which consists of the determination of the intensity distribution of a heat source from the temperature at the boundary of a semiinfinite medium, and of the determination of the corresponding temperature field.

The study of the thermal phenomena, which occur when electromagnetic and acoustic fields interact with various continuous media, is of a considerable scientific and practical interest because of the possibility of increasing the efficiency of exploration and development of oil deposits [1], obtaining unique information by nondestructive methods [2], drying capillary-porous materials [3], passage through frozen soils [4], etc.

The results of experimental studies show [3] that, in addition to the fact that the vhf electromagnetic and acoustic fields separately promote the drying process, they also complement each other. The combination of acoustic interaction with vhf heating increases the drying rate by 30-35% in comparison with the drying process without an acoustic field [3].

At the present time, experiments on the hf electromagnetic and acoustic interactions have been carried out using the model of productive oil-bearing layer [5]. It was noted in [5] that the increase of the hf heat and mass transfer in an acoustic field can clearly be explained by the fact that the dependence of the hf heating on the heat-conduction coefficient becomes more pronounced.

It follows from physical arguments that the effect of the joint interaction of the electromagnetic and acoustic fields should manifest itself by a change in the characteristics of the layer, e.g., in the heat-conduction coefficient, dielectric permeability, and the tangent of the angle of dielectric losses. In particular, as the dielectric permeability and the tangent of the angle of dielectric losses depend not only on the frequency but also on temperature and the density of the medium and, as is known [6, 7], in an intense acoustic field the density of the medium and the temperature change, one should expect a change of parameters of the electromagnetic field.

It is difficult to theoretically describe the effects associated with the joint propagation of hf electromagnetic and acoustic waves, and to study the space–time temperature distribution. This is because in the calculation of the temperature fields, one needs to introduce the intensity of thermal sources, which is determined by the distribution of the electromagnetic field in the medium. It should be noted that to find the distribution of the electromagnetic field is extremely difficult due to the absence of a solution of the Maxwell equations which includes the dependence of the electric properties on the parameters of the intense acoustic field. In other words, to determine the quantity of heat released by a unit volume per unit time, it is necessary to solve a system of coupled equations of thermodynamics, hydrodynamics, and electrodynamics which include the equations of continuity, equations of motion, the energy conservation, equation of state, and the Maxwell equations.

It should also be noted that, at the present time, there is no general theory of wave interaction (with or without taking into account dispersion) in saturated porous media which are the physical analogues of the productive oil-bearing layer.

A number of works [8-11] analyzed some problems of interaction of the electromagnetic and temperature fields in dielectric media which are caused by the temperature dependence of the parameters of the medium. For example, Rikenglaz [8] found the electric and temperature fields in a semiinfinite dielectric with small losses and arbitrary temperature dependences of the parameters of the medium, on which there is incident a plane monochromatic vhf wave. The particular case of linear temperature dependence of the damping coefficient

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of the electromagnetic field (the remaining parameters of the medium being constant) was discussed in [9].

Common to all works [8-11] is the assumption that the heat conduction of the medium is negligibly small, i.e., the adiabatic approximation.

To investigate some features of the interaction of hf electromagnetic and acoustic fields with saturated porous media, it is necessary to include the contribution from the heat transfer by heat conduction. It is also of interest to solve the inverse problems, taking into account the above perturbations.

In the present work we solve the inverse problem which consists of determination of the intensity distribution of a heat source from the temperature at the boundary of the layer, i.e., at the face, and of the corresponding temperature field in a dielectric with small losses which fills the semiinfinite space $x \geq 0$. Examples of these media are oil- and water-saturated rocks, bituminous slates, sulfur, etc.

When there is no interaction between the electromagnetic, acoustic, and temperature fields, i.e., in the linear approximation, the transformation of the energy of the hf electromagnetic waves into the thermal energy results in the decay of the power density of the plane electromagnetic wave according to

$$P(x) = \frac{1}{2} \operatorname{Re}[\bar{\mathbf{E}} \cdot \bar{\mathbf{H}}^*] = \frac{E_0^2}{2z} \exp(-2\alpha x). \tag{1}$$

Here $\bar{\mathbf{E}}, \bar{\mathbf{H}}$ are the intensities of the electromagnetic fields. The asterisk denotes the complex conjugate of a vector. For dielectrics with small losses, $\alpha = \omega\sqrt{\varepsilon}\operatorname{tg}\delta/\lambda_0$; E_0, amplitude of the intensity of the monochromatic electromagnetic field at $x = 0$; ω, cyclic frequency; z, wave resistance of the medium; and λ_0, wavelength of the electromagnetic waves in vacuum.

It can be shown from the solutions of the Maxwell equations that, in general, the heat released in a unit volume per unit time during the interaction of the hf electromagnetic and acoustic fields with a dielectric with small losses, taking into account the temperature dependence of the electrical parameters of the medium, is given by the expression

$$q(x, t) = 2\alpha \frac{E_0^2}{2z} \exp(-2\alpha x)\varphi(t). \tag{2}$$

The function $\varphi(t)$ is subject to determination.

In accordance with expression (2), the temperature distribution in the medium is characterized by an equation of the form

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} + \frac{q}{c\rho}, \quad t \geqslant 0, \ 0 \leqslant x < \infty. \tag{3}$$

This equation will be solved under the following conditions:

$$T(0, x) = T_0, \tag{4}$$

$$T(t, \infty) = T_0, \quad \frac{\partial T(t, \infty)}{\partial x} = 0, \tag{5}$$

$$\frac{\partial T(t, 0)}{\partial x} = 0, \tag{6}$$

$$T(t, 0) = f(t). \tag{7}$$

In expressions (3)-(7), f(t) is a given function in the class C^1, and $f(0) = T_0$.

Thus, we have an inverse problem which consists of determination of the intensity of a heat source as a result of hf heating in an acoustic field, from a given temperature at the boundary of the layer which is measured experimentally. It is known that similar problems are not correctly formulated. However, this is not important for our purposes.

If $\varphi(t)$ is known, the solution of the direct problem (2)-(6) can be written in the form

$$T(t, x) = \int_0^t d\tau \int_0^\infty G(t-\tau, x, \xi)\frac{\alpha E_0^2}{zc\rho}\exp(-2\alpha\xi)\varphi(\tau)\,d\xi, \tag{8}$$

where

$$G(t,\ x,\ \xi)=\frac{1}{2\sqrt{\pi at}}\left\{\exp\left[-\frac{(x-\xi)^2}{4at}\right]+\exp\left[-\frac{(x+\xi)^2}{4at}\right]\right\}$$

is Green's function of the second boundary problem for the heat-conduction equation at a semiaxis.

By putting in (8) x = 0 and using (7), we obtain

$$\int_0^t \varphi(\tau)\,d\tau\int_0^\infty \frac{1}{\sqrt{a\pi(t-\tau)}}\exp\left[-\frac{\xi^2}{4a(t-\tau)}-2\alpha\xi\right]d\xi=f_1(t),$$

$$f_1(t)=[f(t)-T_0]\cdot\frac{zc\rho}{\alpha E_0^2}.$$

We evaluate the inner integral and obtain the following integral equation of the first kind for the function φ(t):

$$\int_0^t K(t-\tau)\,\varphi(\tau)\,d\tau=f_1(t) \tag{9}$$

with the kernel $K(t)=\exp(4a\alpha^2 t)\,\mathrm{erfc}(2\alpha\sqrt{at})$. the solution of this equation is easily found by means of Laplace transform, and has the form

$$\varphi(t)=\frac{zc\rho}{\alpha E_0^2}\left[f'(t)+\frac{2\alpha\sqrt{a}}{\sqrt{\pi}}\int_0^t\frac{f'(\tau)}{\sqrt{t-\tau}}\,d\tau\right]. \tag{10}$$

We shall now clarify which natural conditions must be satisfied by the functions φ(t) and f(t). Clearly, at the initial moment of time (t = 0) we have the following relation

$$q=\alpha\frac{E_0^2}{z}\exp(-2\alpha x). \tag{11}$$

Consequently, in view of (1), we must put $\varphi(0) = 1$. From (10) we then find $f'(0)=\alpha E_0^2/zc\rho$. In addition, f(0) = T_0. These two requirements are satisfied by, e.g., a function of the form

$$f(t)=T_0+\frac{\alpha E_0^2}{zc\rho}t+\psi(t)\,t^2,$$

where ψ(t) is an arbitrary smooth function.

In the simplest case when $\psi(t)\equiv 0$, we have

$$f(t)=T_0+\frac{\alpha E_0^2}{zc\rho}t, \tag{12}$$

and the corresponding φ(t) will be

$$\varphi(t)=1+\frac{4\alpha\sqrt{at}}{\sqrt{\pi}}. \tag{13}$$

In general, T(t, x) can be found from the formula

$$T(t,\ x)-T_0=\frac{\alpha E_0^2}{2zc\rho}\int_0^t \exp[4a\alpha^2(t-\tau)]\left\{\exp(-2\alpha x)\,\mathrm{erfc}\left[\frac{x-4a\alpha(t-\tau)}{2\sqrt{a(t-\tau)}}\right]-\exp(2\alpha x)\,\mathrm{erfc}\left[\frac{x+4a\alpha(t-\tau)}{2\sqrt{a(t-\tau)}}\right]\right\}\varphi(\tau)\,d\tau, \tag{14}$$

where φ(t) is given by (10).

TABLE 1. Increase of the Reduced Temperatures Θ_1 and Θ_2 (°C/(W/m²)) at the Boundary of the Layer as a Function of the Extinction Coefficient of the Electromagnetic Wave (m^{-1})

t,h	$\alpha=0,016\,m^{-1}$			$\alpha=0,1\,m^{-1}$		
	$\Theta_1\cdot10^{-6}$	$\Theta_2\cdot10^{-6}$	δ, %	$\Theta_1\cdot10^{-4}$	$\Theta_2\cdot10^{-4}$	δ, %
2,4	0,263	0,262	0,19	1,104	0,102	2,0
12	2,93	2,92	0,31	1,17	1.14	2,8
23	8,29	8,25	0,53	3,39	3,27	3,9
120	94,6	93,3	1,4	40,23	36,97	8,8
240	270,9	265,6	2,0	118,45	105,33	12,4

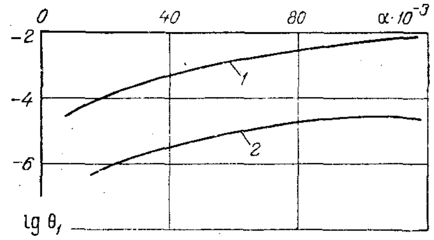

Fig. 1. Logarithm of the reduced temperature as a function of the extinction coefficient of the electromagnetic wave in the medium for the following distances from the source: x = 0 (curve 1) and 2 m (curve 2). The quantity $\alpha \times 10^{-3}$ is plotted in units m^{-1}.

Applying the usual methods [12] one can show that formula (10) is indeed a solution of Eq. (9). Calculations of reduced temperatures $\Theta_1=[T(t,\ x)-T_0]/(2E_0^2/z)$ from formula (14) under condition (13) and Θ_2 for the case $\varphi(t) = 1$ were carried out on a computer, with applications to oil- and water-saturated sandstones with parameters $C\rho$ = 653 kcal/m³ • °C, $a=3.675\cdot10^{-3}$ m²/h [13], and for extinction coefficients of the electromagnetic wave equal to 0.016, 0.036, and 0.1 m^{-1}.

The increase of temperatures Θ_1 and Θ_2 at the boundary of the layer, i.e., for x = 0, which are given in Table 1 shows that the contribution of the effects of interaction of the electromagnetic and acoustic fields in the medium increases with time. The relative increase of the temperatures $\delta=(\Theta_1-\Theta_2)/\Theta_1$ is larger for larger extinction coefficient of the electromagnetic wave. For example, for α = 0.1 m^{-1} and t = 240 h, the quantity δ is 12.4%, but for α = 0.016 m^{-1}, it is only 2%.

Figure 1 which shows the dependences log $\Theta_1 = f(\alpha)$ for x = 0 and x = 2 indicates that, with increasing α, the difference between the curves increases. This agrees with the experimental data [14], and in particular with the fact that at the boundary of a layer in an acoustic field, the temperature is somewhat lower than when the layer is absent. In the depth of the layer, however, it is somewhat higher.

Thus, the results of theoretical investigations confirm that one of the physical reasons for the above features of the interaction of electromagnetic and acoustic fields with dielectrics is the decrease of the extinction coefficient of the electromagnetic wave in the acoustic field (i.e., the electromagnetic-acoustic effect).

The results of this study can be used to justify the mechanism of the joint interaction of the electromagnetic and acoustic fields with dielectric media.

NOTATION

T, temperature, °C; x, distance, m; P, power density of a plane electromagnetic wave, W/m²; α, extinction coefficient of the electromagnetic wave, 1/m; ε, relative dielectric permeability; tan δ, tangent of the angle of dielectric losses; a, effective thermal conductivity of the medium, m²/sec; c, specific heat, J/m³ • °C; ρ, density, kg/m³; G, Green's function; δ, relative temperature increase; t, time, sec; and Re(f(x)), real part of the complex function f(x).

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